

EVALUATING THE DYNAMIC INTERACTION OF  
VEHICLE CHARACTERISTICS AND THE ROAD CONDITION

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SUMMARY

One of the most important developments in pavement engineering was the U.S.A.'s A.A.S.H.O. Road Test, three decades ago. This project provided a comprehensive data base on structural damage and load equivalencies; its results, including the fourth-power law describing the effect of increasing load on pavement condition, have been applied world-wide. Since then, tyres, suspensions, axles and vehicle bodies have been designed which include load-transmitting components that have never been characterised in terms of their effect on the roads. This paper reviews the original criteria, and their relation to recent developments and actual field conditions is evaluated. A proposed research programme is presented. The paper concludes that proper load/damage equivalencies, and possibly even entirely different criteria, must be developed, to more accurately represent the dynamic interaction of modern road transport vehicles and pavements.

INTRODUCTION

The trucking industry operates in a highly competitive environment. As the industry endeavours to increase productivity and lower costs, a natural tendency is to operate vehicles of increasing size and weight, to use tyres that wear less or reduce rolling resistance, and to employ other innovative equipment or methods. Also, the roading authorities should be continually seeking ways to enhance the viability of the industry by altering heavy truck weight, dimension, and tyre-suspension regulations, while simultaneously protecting the investment in the roading infrastructure. This goal can be achieved only if the dynamic interaction of vehicles and road pavements is better understood.

The dynamics of vehicle/pavement interaction is one of the most important, but also least understood, aspects of pavement engineering. Ideally, all of the different loads that will be imposed upon the pavement structure should be assessed, determining the effect of each load on the total available strength of the pavement. Because this is virtually impossible, it is far more simple to express the traffic loading in terms of the number of applications of a standard reference load. For most pavement response models, the wide spectrum of actual axle weights, configurations and spacings between axles is conveniently indexed to one selected standard reference load, usually referred to as an Equivalency Factor. In New Zealand, the equivalency factor is called the Equivalent Design Axle, EDA. This paper discusses the background to the existing criteria, recent developments in coupling vehicle dynamics and road condition, and the need for scientific investigation of dynamic pavement-vehicle interactive behaviour that embodies pavement and vehicle characteristics prevalent in New Zealand.

## BACKGROUND

The function of a road is to safely, comfortably and efficiently serve the users for overall minimum costs to the users and the roading authority. Pavements should be designed and maintained to achieve this function. The two broad categories of paved road structures are rigid pavements, which have Portland Cement Concrete surface layers, and flexible pavements, which have bituminous surfaces. The former are rare in New Zealand and many other countries; thus, this paper concentrates on the latter.

### Pavement Serviceability Concept

The primary operating characteristic of a pavement is the level of service it provides to the users at a certain time. It is important to:

- a) measure and evaluate the level of service to establish the current status of a pavement, and
- b) predict the change in level of service in the future, for either an existing pavement or for a pavement to be constructed, under the cumulative effects of traffic and environmental factors.

The rate of change in the level of service, or serviceability, depends on factors such as axle loads, tyre types and pressures, type and thickness of the pavement, surface distress, original construction quality, climatic factors, and maintenance performed. The history or trend of serviceability over time is called the pavement's performance. Figure 1 shows a generalised performance curve for a typical pavement.

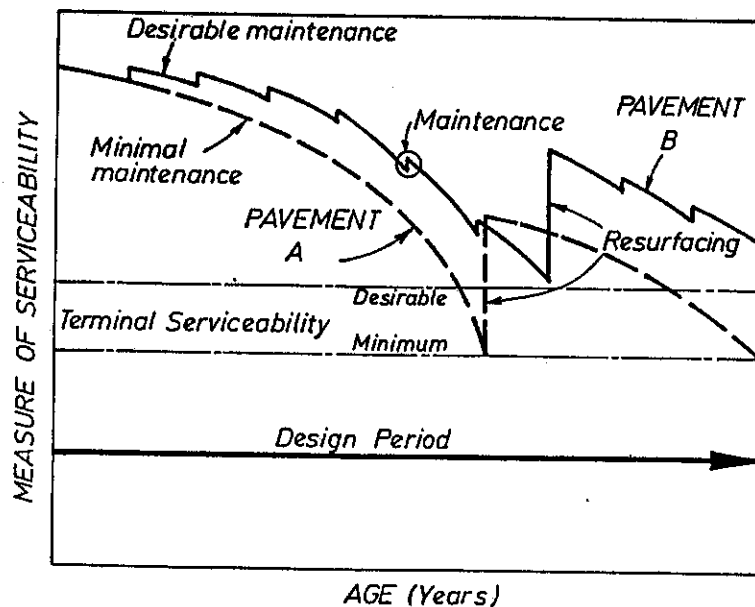


Figure 1 - Pavement Performance

The concept of desirable performance varies greatly amongst pavement engineers. For example, if two engineers were asked to design a pavement for a predicted traffic loading and a 25 year analysis period, one might consider the pavement's performance acceptable only if there was not a single pothole occur during the 25 years, whereas the other might be satisfied if the last predicted vehicle was able to pass safely over the pavement before total collapse at the end of the twenty-fifth year.

Three decades ago, the evaluation of pavement performance was considered inadequate. A pavement was either satisfactory or unsatisfactory (i.e., in need of repair or replacement). Most pavement design concepts in general use did not consider the level of performance desired. The concept of serviceability was developed in conjunction with the AASHO (American Association of State Highway Officials) Road Test by Carey and Irick (1960).

### **AASHO Road Test**

The AASHO Road Test was an expensive (\$US 27 million, 1960) project undertaken to study a number of pavement design factors, during 1958-1961. The test sections, consisting of six loops, were located near Chicago, Illinois. The test included flexible pavements, rigid pavements, and short-span bridges.

In the flexible pavement test sections, the surface course was a bituminous mix; the base course, a well-graded crushed limestone; and the subbase, a uniformly graded sand-gravel mixture. Major design factors were surface, base, and subbase thicknesses. The test sections' surface thickness ranged from 25 to 150 mm, base thickness from 0 to 180 mm and subbase thickness from 0 to 400 mm. Only one subgrade soil, a silty clay, was used in the pavements.

Test traffic included both single and tandem axle vehicles with 10 different axle arrangement - axle load combinations. Single-axle loads ranged from 9000 to 14,000 kg, and tandem-axle loads, from 11,000 to 22,000 kg. Each pavement section was subjected to up to a million load repetitions of two of the possible ten combinations. The AASHO Road Test is the most comprehensive pavement experiment conducted until now. It yielded a wealth of data that has heavily influenced pavement engineering.

### **Axle load - Pavement Response Relationship**

Traffic is usually considered in terms of axle load repetitions. When an axle passes over a point on the road, the subgrade and pavement deflect, then rebound, both underneath and for some distance away from the tyres. Most of the strain is recovered - some immediately, some over a period of time. However, a fraction of the strain is permanent. These small residual strains accumulate under axle passes, resulting in permanent deformation in the subgrade and pavement layers. Many pavement design procedures determine the pavement layer thicknesses required to limit the critical stresses or strains below some specified levels. Deterioration will occur if the pavement is overstressed, but little information was available prior to the early 1960's to relate such deterioration to the pavement's level of service. Since then, the serviceability concept has been adopted by authorities world-wide.

The Palmgren-Miner relationships (Miner's law) are based on research and observations of actual pavements under vehicle traffic. Miner's law is

that the cumulative damage incurred in a pavement subjected to repetitive loadings of the standard design axle can be expressed as follows: The damaging effect ( $\delta$ ) caused by the passage of one axle is proportional to the strain ( $\epsilon$ ) created by that particular loading, or

$$\delta \propto \epsilon$$

If all the axle loads were the same, then the capacity of the pavement, before the maximum allowable damage is reached, can be expressed in terms of N, which is the total number of loading cycles needed for strain  $\epsilon$  to produce failure in the material:

$$\text{Total Allowable Damage (D)} = N\delta, \text{ or } \delta = D/N$$

(D may be rut depth, or cracking or ride roughness; D is specified by the roading authority or design standards).

In any case, D can be expressed as unity, thus,  $\delta = 1/N$ . The effects of each of the various axle loadings could be analysed in this manner, and the results aggregated to determine the total wearing effect.

A means of equating the various loads to one common factor is necessary. One definition of this Equivalency Factor (EF) is:

$$EF = \frac{\text{Number of repetitions to failure using standard axle}}{\text{Number of repetitions to failure using actual axle}}$$

In this case, a pavement has failed when its condition is at or below the minimum acceptable serviceability rating. In some models, the equivalency factor for a vehicle's axles is determined using strain, stress or deflection at any given point in a pavement structure. The current New Zealand model uses multi-layer linear elastic theory for calculating the values.

#### Multi-layer Linear Elastic Theory

Generally, the models used in the theory assume that the pavement materials' properties can be characterised by linear elastic, homogeneous and isotropic behaviour. The critical strains, stresses and deflections for a variety of moduli for subgrade soils, granular layers and bound layers are found by using a computer program. The original version of the most popular program, called BISTRO (BITumen STRUCTures in ROads), was developed by Peutz et al (1968). Numerous other programs are widely used; finite element programs exist but are less popular.

### Fourth-Power Law

The results of the AASHO Road Test were analysed with a complex regression technique, which produced a suite of formulae for calculating pavement thicknesses and load equivalency factors. Subsequently, engineers have simplified the procedure, usually without regard for the original assumptions and application. One of the most important simplifications has been the "Fourth-power Law". The concept can be represented by the formula:

$$\frac{A}{B} = \left[ \frac{L}{S} \right]^y$$

where A is the number of load repetitions of magnitude S, and

B is the number of load repetitions of magnitude L.

By combining all the equivalency factors for rigid and flexible pavements calculated from the AASHO Road Test results, one value for the exponent was found to be 4.15 (AASHO, 1962). This exponent has been modified by various roading authorities, and '4' is the value most commonly used. Thus, the relationship is referred to as the "fourth-power law".

### Derivation of New Zealand's Equivalency Factor

New Zealand's wheel load model is shown in Figure 2. It is based on the wheel load condition commonly used at the time of the AASHO Road Test. The tyre dimensions represent a modified loading area under a 10.00 x 20, 12-ply tyre. The dual-tyred single axle loaded to 80 kN (8.2

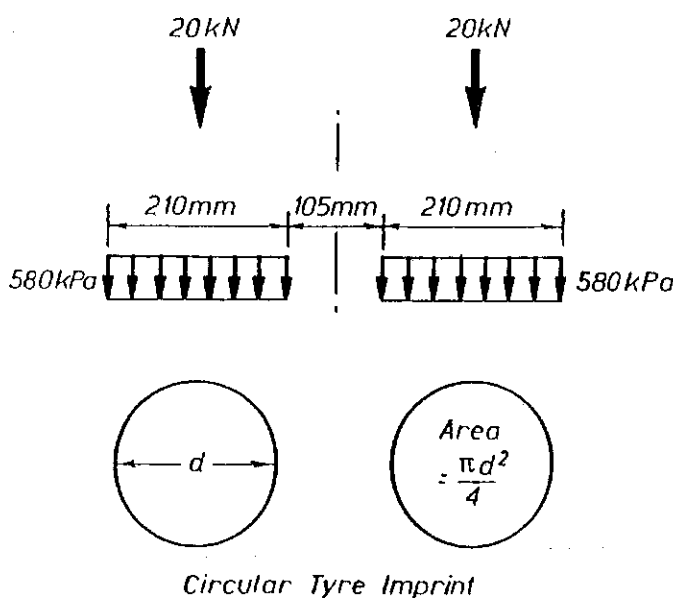


Figure 2 - Loading Model

tonnes) was chosen as the standard axle which causes one unit of wearing effect during one passage over a point. This was the most common loading condition in the U.S. in the 1950's. The load is assumed to always be evenly distributed between the tyres in an axle group.

The term EDA (Equivalent Design Axle) is used in New Zealand as the equivalency factor term. The basic equivalency formula for translating axle loads into EDA is:

$$EDA = \left[ \frac{\text{Weighed Axle Load}}{\text{Reference Axle Load}} \right]^4$$

where the reference axle load

= 6.7 tonnes (65 kN) for a spaced single tyred single axle.

= 8.2 tonnes (80 kN) for a spaced dual tyred single axle.

The multi-layer linear elastic theory is employed for relating other vehicle characteristics, such as tyre type and axle load, to the standard EDA.

#### RECENT DEVELOPMENTS

For many years, the "serviceability rating" was used as a scale for pavements. This index is rather arbitrary, and a "good" value depends largely on the personnel and the equipment used. This problem could be minimised if some absolute standard was available, but one is not. Therefore, "scaling factors" have been developed to provide a basis for comparing ratings from many sources. The evaluation of road quality is a complex problem, depending on three separate components: the pavement user, the vehicle and the pavement roughness, plus interactions among them.

Hutchinson (1964) first described the problems associated with analysing the subjective experience of highway users in deriving an absolute measure of the road quality. Two main requirements are:

- 1) the development of a suitable mathematical model to characterize pavement roughness, and
- 2) the development of a suitable mathematical model to describe the suspension characteristics of highway vehicles that may be used along with the roughness model to predict the dynamic response of vehicles.



## Road Roughness and Dynamic Load Analysis

The road vehicle is a dynamic system composed of masses, springs and damping devices. Thus, its dynamic response is complex, and varies because it is influenced by other parameters such as vehicle speed and road profile. For example, consider only the suspension characteristics. The dynamic relationship between pavements and vehicles can be described as a positive feedback loop: as the heavy vehicles bounce over the road, the higher dynamic load results in greater deformation in the road and its roughness increases, thus increasing the bounce in the vehicles. However, a vehicle with a certain suspension may cause less stress in the pavement than another suspension, but, currently, an accurate model does not exist which can account for this. The coefficient of impact ( $I_c$ ), which is the ratio of dynamic load to static load, is one concept used to quantify the dynamic interaction of pavements and vehicles. The dynamic loads fluctuate in magnitude by up to 50 per cent above the static load. (OECD, 1988).

Another major factor is the tyre/pavement contact pressure. A general, and incorrect, assumption is that the contact pressure in the pavement surface equals the tyre inflation pressure. In reality, the peak contact pressure may be three or more times the inflation pressure.

A number of studies have investigated the relationships between dynamic heavy vehicle wheel loads and pavement response. The most recent and comprehensive have been documented by Sweatman (1983) and Cebon (1988). However, the former still used the "fourth power law" in relating the relative wearing effect of different wheel loads, whereas Cebon has proposed four new criteria for assessing the effect of dynamic wheel loads on pavement behaviour. The criteria are

- (i) aggregate force,
- (ii) fatigue weighted contact stress,
- (iii) fatigue weighted tensile strain, and
- (iv) permanent deformation.

Criteria (i) and (iv) are most relevant to New Zealand conditions. The aggregate force is the combined effect of all wheels and axles on a segment of road. The mean value of a vehicle's aggregate force is approximately equal to the gross vehicle weight, and its variance depends on the dynamics of the vehicle, the road's roughness and the coupling and spacing of its axles. The criteria will be applied in the analyses of a research project currently being conducted at the Transport and Road Research Laboratory (TRRL), U.K.

During the project, an instrumented pavement testing track will be subjected to a variety of loads by an articulated vehicle. The wheel forces and pavement responses experienced under all the axles will be measured simultaneously. However, these studies are only the starting point, and involve pavements very different from New Zealand's.

#### Load Equivalency Exponent

Australian, European and North American studies have demonstrated that the load equivalency exponents for flexible pavements depend very much on the type of pavement failure mechanism (eg. rutting, fatigue cracking, or loss of serviceability), type of pavement (eg. unbound granular basecourse, cemented basecourse or asphaltic concrete), and tyre-axle arrangement (eg. number of tyres and axles, axle spacing, or tyre type). For example, Table 1 shows that the load equivalency exponents for single axles over flexible pavements ranges from 1 to 8, with a concentration of values between 2 and 6 (OECD, 1988; Kinder & Lay, 1988). However, these tests have considered only a limited number of vehicle characteristics, and the flexible pavements tested usually contain an asphaltic concrete structural surface or bound basecourse layers, both of which are uncommon in New Zealand highways.

Table 1 Load Equivalency Exponents for Flexible Pavements

<u>Country of Research</u>	<u>Criterion</u>	<u>Exponent</u>	
Australia	Cracking	2	dual and single-tyre
	Rutting	3.3 - 6	single axles
Finland	Fatigue Cracking	3.3	dual tyre, single axles
"	" "	4	dual tyre, tandem axles
France	Fatigue Cracking	2	dual tyre, single axle
"	Rutting	8	" " " "
Italy	Fatigue Cracking	1.2 - 3	15 axle tyre configurations
USA	Serviceability	4.4	single axles
	Loss	4.9	tandem axles
	Rutting	4.2	single axles
		4.8	tandem axles
	Cracking	1.3 - 1.7	single axles
		1.9	tandem axles

For New Zealand's highways, it has been suggested that the fourth power relationship may be appropriate for pavements surfaced with structural layers of asphaltic concrete, whereas an exponent greater than 4 might be more appropriate for those containing cemented base/subbase layers, and an exponent less than 4 might be more appropriate for unbound pavements, implying a decreased sensitivity to heavy loadings. However, for the sake of simplicity, the fourth power has been used in all cases by the National Roads Board.

### Tyre Pressures and Types

Numerous tests have been conducted into the effect of tyre inflation pressure on pavements. It is widely believed that increasing tyre pressure results in increased stresses and strains within the pavement, but no definitive results are available yet. The most recent comparison of tyre pressure effects suggests that tyre pressure does not significantly influence the equivalency factor for a particular tyre type and load, as shown in Figure 3 (Bonaquist et al, 1988).

One of the most important changes during the last two decades has been the increasing use of radial tyres and wide-base tyres. Figure 3 (Bonaquist et al, 1988) illustrates that, at higher than legal loads and higher tyre pressures, the detrimental affect of bias tyres on the fatigue of a flexible pavement is greater than that of radial tyres.

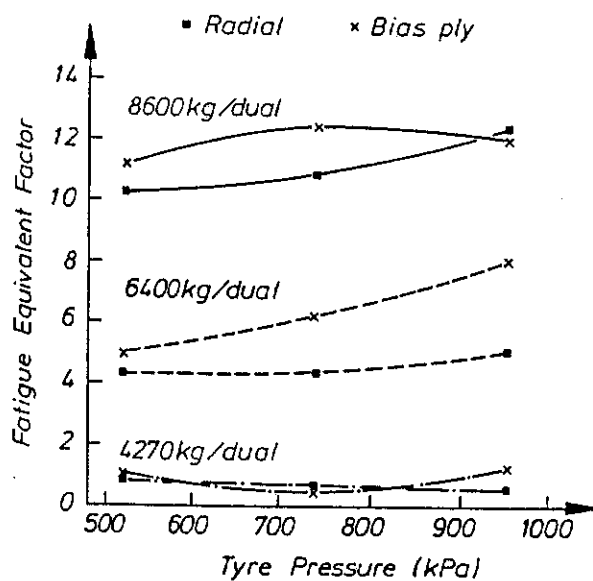


Figure 3 - Equivalency Factors for Different Loads  
Tyre Types and Tyre Pressures  
(Criteria: Fatigue in Asphaltic Concrete) (after Bonaquist et al)

Papagiannakis and others (1988) collated research findings about the equivalent loads of single wide-based tyres and normal dual tyres. The loads vary significantly depending on the criteria used to compare the effects, as shown in Table 2.

Table 2 - Equivalent Loads of Single Wide-based Tyres

EVALUATION METHOD	CRITERION	LOAD EQUIVALENCY		RESEARCHER	
		DUALS (kN)	SINGLE WIDE-BASE (kN)		
Benkelman Beam	Deflection	80	57	Zube	1965
Benkelman Beam	Deflection Bowl	80	50-55	Emery	1967
Theoretical	AC Strains	80	48-63	Deacon	1969
Theoretical	AC Strains	80	56	Terrel	1976
Instruments	AC Strains-Defl.	80	64-75	Christison	1978
Instruments	AC Strains	80	44	Battiato	1978
Instruments	AC Strains-Defl.	98	49-74	Christison	1980
Instruments	Deflection	80	61-75	Snelgrove	1980
Theoretical	Work Strain	80	66	Southgate	1985
Instruments	Deflection	80	59	Sharp	1986

AC = Asphaltic Concrete

(after Papagiannakis et al, 1987)

#### Pavement Materials Characterisation and Behaviour

Contrary to the major assumptions of the multi-layer linear elastic theory and computer program discussed earlier, in reality, the behaviour of materials most commonly found in New Zealand's pavements tend to be non-linear, elasto-plastic, very non-homogeneous and definitely anisotropic. For example, the modulus of a granular basecourse layer depends on the stresses induced in that material by the load being applied; this is non-linear behaviour. Also, the properties of adjacent samples of the material can be dramatically different; it is non-homogeneous. And the stresses and strains depend on direction; they are anisotropic. The program's input is limited to the model characteristics shown in Figure 2, such as a circular loaded area and uniform contact pressure. A suitable model would allow for different characteristics and dynamics of the

suspensions, tyres and axles, such as tyre construction, stiffness, footprints and inflation pressures, and the sprung/unsprung masses.

#### FUTURE RESEARCH NEEDED

The AASHO Road Test report's recommendations emphasised that "the findings of the AASHO Road Test relate specifically to the physical environment of the project, to the materials used in the pavements, to the range of thicknesses and loads and number of load applications included in the experiments, to the construction techniques employed, to the specific times and rates of application of test track, and to the climatic cycles that occurred during construction and testing of the experimental pavements. Generalisations and extrapolations of these findings to conditions other than those that existed at the road test should be based upon experimental or other evidence of the effects on pavement performance of variations in climate, soil type, materials, construction practices and traffic." (AASHO, 1962)

A great deal of confusion still exists as to the load equivalency factors appropriate to New Zealand conditions. The first phase of resolving the problem is to identify the issues. Two separate, but interdependent, issues involve determining load equivalencies of different vehicles for input to:

1. models for pavement design and predicting pavement performance, and
2. models for comparing relative effects of vehicle characteristics.

The load equivalency factors for a pavement design model are not so critical. Pavement design is not very sensitive to the number of EDAs because other aspects, such as environmental and moisture conditions, have a much greater influence. Research carried out in New Zealand on typical pavements indicates that, if the aggregate used in the granular layers possesses sufficient strength to resist traffic loadings and the drainage is adequate, then loss of serviceability will be due primarily to subgrade deformation and not aggregate deterioration. In other words, good quality aggregate should survive an infinite number of load

repetitions without appreciable deterioration. Also, loading conditions will change over the pavement's life regardless of the original model.

Load equivalencies of different vehicles for input to models for predicting pavement performance after construction should be improved. In order to reduce the users' and the roading authorities' costs, pavement management practices are being implemented for programming maintenance and rehabilitation. A complete pavement management system coordinates planning, design, construction, maintenance, rehabilitation, research and information exchange so as to minimise costs and disruptions to the various road transportation sectors. Once Weigh-in-Motion devices are installed throughout the road network, most of the vehicle loads will be measured. Using the accurate data on vehicle loads, the pavement's actual performance can be compared with the predicted. The pavement performance model will be used to predict when deterioration reaches a critical level, so that preventative maintenance can be scheduled and, thus, costs are reduced. Normally, by the time pavement deterioration manifests itself as a pothole or a rut, the remedy is quite expensive.

The second issue, the axle weights and loading model, requires substantially more research, so as to determine load equivalency factors for allowable axle weights for different tyres and vehicles and for a road user charges allocation model. In this case, the linear elastic theory is unsuitable for evaluating the relative effects of different vehicles. Cebon's models are reasonably good for comparing some characteristics of different vehicles, but can not account for many of the pavement factors. Also, this issue also must take into account whether the major pavement criteria is:

- i) incremental damage, which is related to level of service, or
- ii) total pavement life, which is related to performance.

#### Research Needed

Because of New Zealand's unique situation, with respect to both the road user charges incurred by the road transport industry and the dependence on thin-surfaced flexible pavements, a major research effort is needed. The investment in our roading infrastructure is worth at least fifty billion dollars and the road transport sector contributes to or affects one-third of the gross national product. Spending just 0.1 per cent of

the total annual roading expenditure, or \$750,000, per year over the next five years addressing the dynamic vehicle/pavement interaction subject is quite justified, and would yield incredible value. This research would ensure that the road user charges, which cost the road transport industry over \$300 million annually, and other road-related charges are allocated fairly.

The research must isolate the various components of the vehicle/pavement interaction system, such as the static and dynamic components of vehicle loading, and the relative effects of vehicles, the environment and the pavement materials. Laboratory testing and computer analysis alone are inappropriate to the nature of the work involved. Trials utilising full-scale equipment and pavements are necessary. A number of such research projects within New Zealand are underway or in the proposal stage.

One of the projects involves subjecting a series of instrumented pavements, constructed and tested in the Canterbury Accelerated Pavement Testing Indoor Facility (CAPTIF), to a variety of tyre pressures, tyre types and numbers, and loads. The literature review and preliminary work are completed, and the testing phase is scheduled for mid-1989. The research is being done as part of a PhD study by the author, and is funded jointly by the National Roads Board, the University Grants Committee, and the University of Canterbury.

Subsequent projects proposed will take advantage of CAPTIF's versatility, and would involve studying the interaction of pavements and tandem axles, tri-axle arrangements, and various tyre and suspension parameters. Also, environmental simulation systems shall be installed so that the relative effects of climatic influences and vehicle characteristics on pavement behaviour can be evaluated under controlled conditions. Simultaneously, the long-term performance of an instrumented and closely-monitored field pavement must be analysed.

A related project being conducted by the Auckland DSIR-AIDD, and funded jointly by that organisation and the National Roads Board, is assessing heavy vehicle suspensions with respect to road surface profiles. The project is described in another paper presented at this seminar.

These research projects must be properly managed and coordinated so as to maximise the benefits. All sectors of the road transportation field, including transport and pavement engineers, and researchers, should be consulted and be prepared to offer constructive advice. Only one aspect of the research is explored further here.

#### CAPTIF

CAPTIF's primary purpose is the accelerated testing and evaluation of road pavements and subgrades by replicating the effect on the pavement of actual traffic conditions. In addition, a variety of tyre, axle, suspension, braking and loading systems can be tested. The significant feature of this international first-class facility is its simulated loading and vehicle emulator (slave) capable of applying a myriad of loading conditions via an array of vehicle types and assemblies. A differing mix of road speeds, up to 50 km/h, can be attained because the slave may run at a constant speed or any chosen selection of speeds for varying durations. Controlled accelerating, braking and constant speed modes are available, and can be applied to either selected segments of the track or its whole length.

Testing can be conducted with any pair of similar vehicle types or with a different vehicle on each arm. A vehicle consists of half-axles, wheel-driving hydraulic pumps, normal wheels and suspensions, a frame, and instrumentation. A standard rear axle of a truck was split and one-half of the assembly was used for each vehicle. The differential was replaced with a driving motor so that the driving force is produced at the road surface in the same manner as would a conventional vehicle. Standard wheel hubs and heavy duty truck tyres are used. The standard vehicles are equipped with single-axle, dual-tyred wheel assemblies which can be loaded to between 21 and 46 kN, or higher if modified.

#### CONCLUSION

The "fourth power" concept used for comparing the relative damaging effect of different vehicular loading conditions is based on test data with major limitations, obtained three decades ago and altered little since. The concept's main load equivalency conversion exponent (the "fourth-power law") is derived through generalised and simplified



assumptions that still need to be confirmed for New Zealand conditions. Loading and pavement behaviour models that incorporate various aspects, such as dynamic pavement vehicle interaction, tyre types and pressures, and non-linearity of unbound granular layers moduli, should be developed specifically for New Zealand, especially considering the unique system of road user charges.

#### REFERENCES

- "The AASHO Road Test - History and Description of Project", Highway Research Board Special Report 61A, Washington, D.C., 1961.
- "The AASHO Road Test - Report 5 Pavement Research", Special Report 61E, Highway Research Board, Washington, D.C., 1962.
- "Heavy trucks, climate and pavement damage", Road Transport Research, OECD, Paris, 1988.
- Bonaquist, R., Churilla, C. and Freund, D., "Effect of Load, Tire Pressure, and Tire Type on Flexible Pavement Response", Public Roads Vol. 52, No. 1, U.S. Dept. of Transportation, Washington, D.C., June 1988.
- Cebon, D. "Theoretical road damage due to dynamic tyre forces of heavy vehicles, Part 2 : simulated damage caused by a tandem-axle vehicle", Proc. Instn. Mech. Engrs., Vol. 202, No. 1, London, U.K., 1988.
- Kinder, D.F., and Lay, M.G., "Review of the Fourth Power Law", Internal Report 000-248, Australian Road Research Board, Melbourne, May, 1988.
- Papagiannakis, A.T., Haas, R.C.G., and Phang, W.A., "Wide-base Truck Tires; Industry Trends and State of Knowledge of Their Impact on Pavements", Proc. of Roads and Transportation Assoc. of Canada Annual Meeting, Saskatoon, Canada, Sept. 1987.
- Peutz, M.G.F., van Kempen, H.P.M, and Jones, A., "Layered Systems Under Normal Surface Loads", Highway Research Record No. 228, Highway Research Board, Washington, D.C., 1968.
- Schnitter, O., "Comparison of Stresses, Strains and Deflections Calculated with Various Layer Programs", University of Texas at Austin, U.S.A., 1977.
- Sweatman, P.F. "A study of dynamic wheel forces in axle group suspensions of heavy vehicles", Australian Road Research Board Special Report SR27, Melbourne, June 1983.